# OLIMPIADA NAȚIONALĂ DE MATEMATICĂ

Faza locală-10.02.2024

Clasa a XI-a

*Barem de corectare*

**1. a)** $\det A = 2 \neq 0 \Rightarrow$ A inversabilă…………………………………………1p

$A^{-1} = \frac{1}{2}\begin{pmatrix} 2 & -1 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{pmatrix}$……………………………………………3p

**b)** $A = I_3 + \begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} = I_3 + B, \quad I_3 \cdot B = B \cdot I_3$ ……………………1p

$A^n = (I_3 + B)^n = C_n^0 I_3 + C_n^1 B + C_n^2 B^2 + \cdots + C_n^n B^n$ cu $B^p = B, p \geq 1$ ……………1p

$\Rightarrow A^n = I_3 + (2^n - 1)B = \begin{pmatrix} 1 & 2^n - 1 & 2^n - 1 \\ 0 & 2^n & 2^n - 1 \\ 0 & 0 & 1 \end{pmatrix}$……………………………1p

**2.** Tr(X) = Tr(A B – B A) = Tr(A B) – Tr(B A) = 0 ……………………………2p

Fie $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow$ Tr(X) $= a + d = 0$ ……………………………1p

$det$(X) $= a\,d - b\,c = -1$……………………………………………1p

$det(I_2 - X) = \begin{vmatrix} 1-a & -b \\ -c & 1-d \end{vmatrix} = 1$ - Tr(X) + $det$(X) = 0 ……………………2p

$\Rightarrow I_2$ – X nu este inversabilă……………………………………………1p

**3.** $x_n = n(a\sqrt{1+\frac{1}{n}} + b\sqrt{1+\frac{2}{n}} + c\sqrt{1+\frac{3}{n}})$……………………………1p

$\lim_{n\to\infty} x_n = \begin{cases} -\infty, dac\breve{a}\ a+b+c<0 \\ +\infty, dac\breve{a}\ a+b+c>0 \end{cases}$ ……………………………3p

Dacă $a + b + c = 0$,

atunci $x_n = b\left(\sqrt{n^2+2n} - \sqrt{n^2+n}\right) + c\left(\sqrt{n^2+3n} - \sqrt{n^2+n}\right)$ ……………………1p

$\lim_{n\to\infty} x_n = \frac{b}{2} + c$ ……………………………………………2p

**4.** $x_{n+1} + y_{n+1} = x_n + y_n \Rightarrow x_n + y_n = a + b$ ……………………………1p

$x_{n+1} - y_{n+1} = -\frac{3}{11}(x_n - y_n) \Rightarrow x_n - y_n = \left(-\frac{3}{11}\right)^{n-1}(a-b)$……………………2p

$\boldsymbol{x_n} = \frac{\boldsymbol{a+b}}{\boldsymbol{2}} + \frac{\boldsymbol{a-b}}{\boldsymbol{2}} \cdot \left(-\frac{3}{11}\right)^{n-1}$ ..............................................................................1p

$\lim\limits_{n\to\infty} x_n = \frac{\boldsymbol{a+b}}{\boldsymbol{2}} \in \mathbf{R} \Rightarrow (x_n)_{n\geq 1}$ convergent.............................................................1p

$\boldsymbol{y_n} = \frac{\boldsymbol{a+b}}{\boldsymbol{2}} - \frac{\boldsymbol{a-b}}{\boldsymbol{2}} \cdot \left(-\frac{3}{11}\right)^{n-1}$ ..............................................................................1p

$\lim\limits_{n\to\infty} y_n = \frac{\boldsymbol{a+b}}{\boldsymbol{2}} \in \mathbf{R} \Rightarrow (y_n)_{n\geq 1}$ convergent.............................................................1p

Propunători:

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