OLIMPIADA NAŢIONALĂ DE MATEMATICĂ

ETAPA LOCALĂ-VRANCEA

08.02.2020

CLASA a X-a

Barem de corectare

1. $1-3\sqrt[3]{2}+2\sqrt[3]{4}=\left(\sqrt[3]{2}-1\right)\left(2\sqrt[3]{2}-1\right)$…..1p

$\frac{5}{(\sqrt[3]{2}-1)(2\sqrt[3]{2}-1)}-\frac{17}{(\sqrt[3]{2}+1)(2\sqrt[3]{2}+1)}-2\sqrt[3]{2}=$ ……………………………….…..2p

$=\frac{(\sqrt[3]{4}+\sqrt[3]{2}+1)(4\sqrt[3]{4}+2\sqrt[3]{2}+1)-(\sqrt[3]{4}-\sqrt[3]{2}+1)(4\sqrt[3]{4}-2\sqrt[3]{2}+1)}{3}-2\sqrt[3]{2}=$……………….......3p

$=\frac{24+6\sqrt[3]{2}}{3}-2\sqrt[3]{2}=8\in\mathbb{N}$………………...1p

2. Cazul I: $\begin{cases}-\frac{a}{2}\leq 1\\ f(1)=1\\ f(4)=10\end{cases}$ ………………………………………………………........3p

Cazul II: $\begin{cases}-\frac{a}{2}\geq 4\\ f(1)=10\\ f(4)=1\end{cases}$ ……………………………………………………...........................3p

Soluţii: $a=-2$, $b=2$ şi $a=-8$, $b=17$ ……………………………………………….…1p

3. $\sin\alpha=\cos\left(\frac{\pi}{2}-\alpha\right)=\cos x,\ x=\frac{\pi}{2}-\alpha$ 1p

$z+\frac{1}{z}=2\cos x\Rightarrow z^2-2z\cos x+1=0$ 1p

$z_{1,2}=\cos x\ \pm i\ sinx$ ………………………………………………………..2p

$z_1 = \frac{1}{z_2}$ şi $z_2 = \overline{z_1}$ ……………………………………………………………1p

$z^n + \frac{1}{z^n} = z_1^n + z_2^n = z_1^n + \bar{z}_1^n = 2\cos nx = 2\cos n\left(\frac{\pi}{2} - \alpha\right)$ ……………………………………2p

4. Dacă $n = 1 \Rightarrow z = -\frac{a}{2} \in R$, fals $\Rightarrow n \geq 2$ …………………………………………………………1p

$\left.\begin{matrix} z^n + z \cdot n + a = 0 \\ \bar{z}^n + \bar{z} \cdot n + a = 0 \end{matrix}\right\} \Rightarrow z^n - \bar{z}^n + n(z - \bar{z}) = 0$

$z \in$ **C** **R** $\Rightarrow (z - \bar{z}) \neq 0$ ……………………………………………………………………………2p

Deci $-n = z^{n-1} + z^{n-2}\,\bar{z} + \cdots + \bar{z}^{n-1}$ …………………………………………2p

$n = |-n| = |z^{n-1} + z^{n-2}\,\bar{z} + \cdots + \bar{z}^{n-1}| \leq n\,|z|^{n-1} \Rightarrow |z|^{n-1} \geq 1 \Rightarrow |z| \geq 1$ …2p