Profil Științele Naturii

BAREM - Clasa a XII a

I.

1. $\bar{z} - 2z = 4 - 3i$ 2p
   $|\bar{z} - 2z| = 5$ 3p
2. $x + y = 1;\ x^2 + y^2 = 1; xy = 0$ 2p
   $x = 0$ și $y = 1$ 3p
3. $n \in \mathbb{N}; n \geq 3;\ n^2 - 2n - 15 \leq 0$ 2p
   $n \in \{3,4,5\}$ 3p
4. $8 \cdot 2^{2x} - 12 \cdot 2^x + 4 = 0$ 2p
   $2^x = 1;\ 2^x = 2^{-1};\ x \in \{-1; 0\}$ 3p
5. d(A,C) = d(B,C) $\Rightarrow m \in \sqrt{3}$ 5p
6. $\sin\frac{11\pi}{12} = \sin(\frac{4\pi}{6} + \frac{\pi}{4}) = \sin(\pi - \frac{\pi}{12})$ 1p
   $\cos\frac{23\pi}{12} = \cos(2\pi - \frac{\pi}{12})$ 1p
   $\sin\frac{11\pi}{12} \cdot \cos\frac{23\pi}{12} = \frac{1}{4}$ 3p

II. 1.

a) $|A| = 0$ 5p

b) $A^2 = 5 \cdot A$; a = 5 .... 5p

c) $(I_3 + A)\left(I_3 - \frac{1}{6}A\right) = I_3$ 2p

Deci $(I_3 + A)^{-1} = I_3 - \frac{1}{6}A$ 3p

2.

a) Demonstrația 5p

b) (-10)∗(-9)∗(-8)∗...∗(-1) ∗0∗1∗...∗10 = -1.......... 5p

c) a = 0 sau a = -2 .... 5p

III. 1.

a) $f'_{(x)} = 2x - \frac{2}{x} = \frac{2x^2-2}{x} = \frac{2(x^2-1)}{x}$ 5p

b) $d: f = \frac{35}{3}x + \frac{2020}{3}$ 1p

$f'_{(x_0)} = \frac{35}{3}\ cu\ x_0 \in (0, \infty)$ 2p

$\frac{2(x_0^2-1)}{x_0} = \frac{35}{3} \Leftrightarrow 6x_0^2 - 35x_0 - 6 = 0 \Leftrightarrow x_0 = 6; x_1 = -\frac{1}{6} \notin (0, \infty)$ 2p

c) $x \in (0,1);\ f'_{(x)} < 0 \Rightarrow f(x)\ descrescătoare$

$x > 1;\ f'_{(x)} > 0 \Rightarrow f(x)\ crescătoare$

$f(1) = 2 \qquad ld(0) = \lim_{\substack{x \to 0 \\ x > 0}} f_{(x)} = \lim_{\substack{x \to 0 \\ x > 0}} (x^2 - 2\ln x + 1) = +\infty$ 1p

$f(1) = 1 - 2\ln 1 + 1 = 2$ 1p

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} x^2(1 - \frac{2\ln x}{x^2} + \frac{1}{x^2}) = +\infty$ 1p

Din tabelul de variație f nu are rădăcini reale 2p

2.

a) $F(x) = \int f(x)\,dx = \frac{1}{2}\int \frac{2x}{x^2+7}dx = \frac{1}{2}\ln(x^2+7) + C$ 2p

$F(0) = \frac{1}{2}\ln 7 + C \Rightarrow C = 2020 - \ln\sqrt{7}$ 1p

$F(x) = \frac{1}{2}\ln(x^2+7) + 2020 - \frac{1}{2}\ln 7$ 2p

b) $\int_0^{\sqrt{7}} x^3 f(x) = \int_0^{\sqrt{7}} \frac{x^4}{x^2+7}dx = \int_0^{\sqrt{7}} \frac{x^4-49}{x^2+7}dx + 49\int_0^{\sqrt{7}} \frac{1}{x^2+7}dx =$

$= \int_0^{\sqrt{7}} (x^2-7)dx + 49\int_0^{\sqrt{7}} \frac{1}{x^2+(\sqrt{7})^2}dx$ 2p

$= \frac{x^3}{3}\Big|_0^{\sqrt{7}} - 7x\Big|_0^{\sqrt{7}} + \frac{49}{\sqrt{7}} arctg\frac{x}{\sqrt{7}}\Big|_0^{\sqrt{7}} = \frac{7\sqrt{7}}{3} - 7\sqrt{7} + \frac{49\sqrt{7}}{7} arctg$

$= \frac{-14\sqrt{7}}{3} + 7\sqrt{7}\cdot\frac{\pi}{4}$ 3p

c) $f^{`}_{(x)} = \frac{(x)^{`}(x^2+7) - x(x^2+7)^{`}}{x^2+7} = \frac{x^2+7-2x^2}{(x^2+7)^2} = \frac{7-x^2}{(x^2+7)^2}$ 1p

$x \in (0,1);\ f^{`}_{(x)} > 0 \Rightarrow f(x)\ crescătoare$

$f(0) = 0; f(1) = \frac{1}{8}$ 1p

$f(x) \le \frac{1}{8} (\forall) x \in [0,1];\quad \int_0^1 f(x)\,dx \le \int_0^1 \frac{1}{8}dx$ 1p

$\Leftrightarrow \int_0^1 f(x)\,dx \le \frac{1}{8}\int_0^1 dx = \frac{1}{8}x\Big|_0^1 = \frac{1}{8}$ 1p

Deci $\int_0^1 f(x)\,dx \le \frac{1}{8}$ 1p

Notă:

* La orice soluție corectă se acordă punctaj maxim.
Se acordă 10 puncte din oficiu.