MINISTERUL EDUCAŢIEI ŞI CERCETĂRII

Profil Științele naturii

BAREM - Clasa a X a

I. a) $\sqrt[4]{\frac{3+2\sqrt{2}}{3-2\sqrt{2}}}=\sqrt{3+2\sqrt{2}}$ .....5 p

Finalizare .....5 p

b) $(\sqrt{2}-1)^{2020}=(\sqrt{2}-1)^{2018}(\sqrt{2}-1)^{2}=(\sqrt{2}-1)^{2018}(3-2\sqrt{2})$.....4 p

$(\sqrt{2}+1)^{2020}=(\sqrt{2}+1)^{2018}(\sqrt{2}+1)^{2}=(\sqrt{2}+1)^{2018}(3+2\sqrt{2})$.....2 p

Finalizare .....4 p

c) $(\sqrt{2}+1)^{x}+(\sqrt{2}-1)^{x}(\sqrt{2}+1)=2+\sqrt{2}$..... 2 p

$(\sqrt{2}+1)^{2x}+(\sqrt{2}+1)=(2+\sqrt{2})(\sqrt{2}+1)^{x}$ ..... 1 p

$(\sqrt{2}+1)^{x}=t, t>0$.....1 p

$t^2-(2+\sqrt{2})t+\sqrt{2}+1=0$.....1 p

$t\in\{1,1+\sqrt{2}\}$.....3 p

$x\in\{0,1\}$.....2 p

II. a)

i) $a=(\log_2 x)^2\left(1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}\right)$.....3 p

$b=(\log_2 x)^2\left(\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}+\frac{1}{6}-\frac{1}{7}\right)$.....3 p

$a-b=(\log_2 x)^2\left(\frac{2}{3}-\frac{2}{4}+\frac{2}{5}-\frac{2}{6}+\frac{1}{7}\right)>0$.....4 p

ii) $a+b=(\log_2 x)^2\cdot\frac{6}{7}$.....4 p

$(\log_2 x)^2=\frac{1}{4}$.....4 p

$x\in\left\{\frac{1}{\sqrt{2}},\sqrt{2}\right\}$.....2 p

b) Aplicarea proprietății $a^{\log_a x}=x$.....2 p

$A=1+3+5+\cdots+91$.....4 p

$A=46^2$.....4 p

III. a) $z_1\cdot z_2=i\cdot|z_2|^2$.....2 p

$z_1\cdot z_2=\mathrm{i}(\mathrm{a}^2+\mathrm{b}^2)$.....6 p

Finalizare.....2 p

b) $\begin{cases}3a-5b=2\\-5a+3b=-2\end{cases}$.....4 p

$(a,b)=\left(\frac{1}{4},-\frac{1}{4}\right)$.....6 p

c) $(z_1-iz_2)^n=(2a)^n$.....2 p

$(\overline{z_1}-i\,\overline{z_2})^n=(-2ib)^n$.....2 p

$(z_1-iz_2)^n\cdot(\overline{z_1}-i\,\overline{z_2})^n=(-4ab)^n\cdot i^n$.....2 p

$n\in\{4k,4k+2\}, k\in\mathbb{N}^*$.....4 p

* La orice soluție corectă se acordă punctaj maxim.
Se acordă 10 puncte din oficiu.