

Profil Pedagogic

BAREM - Clasa a X a

I.

Pc.	Detalii rezolvare	Barem asociat
a)	$\frac{1}{\sqrt[3]{a}-1} - \frac{1}{\sqrt[3]{a^2} + \sqrt[3]{a} + 1} = \frac{\sqrt[3]{a^2} + \sqrt[3]{a} + 1 - \sqrt[3]{a} + 1}{a-1} = \frac{\sqrt[3]{a^2} + 2}{a-1}$	3p
	$\frac{1}{\sqrt[3]{a} + 1} + \frac{1}{\sqrt[3]{a^2} - \sqrt[3]{a} + 1} = \frac{\sqrt[3]{a^2} - \sqrt[3]{a} + 1 + \sqrt[3]{a} + 1}{a+1} = \frac{\sqrt[3]{a^2} + 2}{a+1}$	3p
	$E(a) = \frac{a+1}{a-1}$	2p
	$E(a) = 3, \text{ natural}$	2p
b)	$E(a) = \frac{a+1}{a-1} = 1 + \frac{2}{a-1}$	3p
	$E(a) \text{ întreg dacă } \frac{2}{a-1} \in \mathbb{Z}$	2p
	$\frac{2}{a-1} \in \mathbb{Z} \Leftrightarrow a \in \{0, 2, 3\}$	5p
c)	Detalii rezolvare	Barem asociat
	$\log_{\sqrt[3]{2}} \left(1 - \frac{1}{5}\right) \left(1 - \frac{1}{6}\right) \dots \left(1 - \frac{1}{512}\right)$	3p
	$\log_{\sqrt[3]{2}} \frac{4}{5} \cdot \frac{5}{6} \cdot \dots \cdot \frac{511}{512} = \log_{\sqrt[3]{2}} \frac{4}{512}$	3p
	$\log_{\sqrt[3]{2}} \frac{4}{2^9} = \log_{\sqrt[3]{2}} 2^{-7}$	2p
	$\frac{\log_2 2^{-7}}{\log_2 \sqrt[3]{2}} = \frac{-7}{\frac{1}{3}} = -21$	2p

II. a)

Detalii rezolvare	Barem asociat
$E(x) = (x^{\sqrt{6}})^{\sqrt{2}} : (x^2)^{\sqrt{3}+2} \cdot \frac{16}{3^2}$	4p
$E(x) = x^{\sqrt{12}} \cdot x^{2\sqrt{3}+4} \cdot \frac{16}{3^2} = \frac{16}{x^4 3^2}$	3p
$E(0, (6)) = \frac{16}{\left(\frac{2}{3}\right)^4 3^2} = 9 < 10$	3p

Pc.	Detalii rezolvare	Barem asociat
b)	$A = \log_x a + \log_x a^2 + \dots + \log_x a^n = (1 + 2 + 3 + \dots + n) \log_x a$ $= \frac{n(n+1)}{2} \log_x a$	3+3+3+1p
c)	$u = \frac{\log_2(2 \cdot 3^2)}{\log_2(2^2 \cdot 3)} = \frac{1 + 2t}{2 + t}$	3p
	$v = \frac{\log_2(2 \cdot 3^3)}{\log_2(2^3 \cdot 3)} = \frac{1 + 3t}{3 + t}$	3p
	$u \cdot v + 5(u - v) = \frac{(1 + 2t)(1 + 3t)}{(2 + t)(3 + t)} + 5 \cdot \left(\frac{1 + 2t}{2 + t} - \frac{1 + 3t}{3 + t} \right)$ $= \frac{t^2 + 5t + 6}{t^2 + 5t + 6} = 1$	4p

III. a)

Detalii rezolvare	Barem asociat
$\left(\frac{2}{5}\right)^{\frac{6-5x}{5x+2}} \leq \left(\frac{5}{2}\right)^{-2} \Rightarrow \frac{6-5x}{5x+2} \geq -2, \text{ cu } x \neq -\frac{2}{5}$	5p
$\frac{6-5x}{5x+2} \geq -2 \Rightarrow \frac{x+2}{5x+2} \geq 0 \Rightarrow x \in (-\infty, -2] \cup \left(-\frac{2}{5}, \infty\right)$	5p

b)

Detalii rezolvare	Barem asociat
$(3^x + 3^{-x})^2 = 49 \Rightarrow 9^x + 9^{-x} + 2 = 49 \Rightarrow 9^x + 9^{-x} = 47$	4p
$(9^x + 9^{-x})^3 = 47^3 \Rightarrow 9^{3x} + 3 \cdot 9^x \cdot 9^{-x} (9^x + 9^{-x}) + 9^{-3x} = 103823$	6p

c)

Detalii rezolvare	Barem asociat
$G_f \cap (Oy) \Rightarrow x = 0 \text{ și } y = f(0) = m^2 - 4m + 5$	2p
Punctul de intersecție este $A(0, m^2 - 4m + 5)$	2p
Lungimea segmentului [OA] este $ OA = m^2 - 4m + 5 $	1p
Dar $m^2 - 4m + 5 > 0$, oricare ar fi $m \in \mathbb{R} (\Delta < 0)$	2p
$ OA = m^2 - 4m + 5$, iar $\min(m^2 - 4m + 5) = \frac{\Delta}{4a} = 1$	2p
Așadar, valoarea minimă a lungimii segmentului [OA] este 1	1p

Notă:

* La orice soluție corectă se acordă punctaj maxim.
Se acordă 10 puncte din oficiu.